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THE EFFECT OF THE SLIPSTREAM ON AN AIRPLANE WING

By A. Franke and F. Weinig

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THE EFFECT OF THE SLIPSTREAM ON AN AIRPLANE WING*

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SUMMARY

The present report is in some respects a continuation of an article on the subject entitled "Influence of the Propeller on other Parts of the Airplane Structures," by C. Koning, published in Aerodynamic Theory (1935), vol. 4, p. 361. This article has been simplifed through the present one in the case of a wing spanning the slipstream and extended to include slipstream rotation and propeller in yaw.

The conditions which must be met at the slipstream boundary are developed; after which it is shown with the aid of the reflection method how these limiting conditions may be complied with for the case of an airfoil in a propeller slipstream in horizontal flow as well as for the propeller in yaw and with allowance for the slipstream rotation. In connection herewith, it is shown how the effective angle of attack and the circulation distribution with due regard to slipstream effect can be predicted and what inferences may be drawn therefrom for the distribution of lift, drag, and pitching moment across the span.

I. INTRODUCTION

A considerable portion of the wing and usually also of the tail of an airplane are directly affected by the propeller slipstream. The larger the proportion of these surfaces and the higher the propeller loading, its coefficient of advance and angle-of-attack range, the greater the slipstream effect on the airplane. However, this effect is not confined to parts within the slipstream but to parts outside of it as well, according to recent find-

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*"Tragflügel und Schraubenstrahl." Luftfahrtforschung, vol. 15, no. 6, June 6, 1938, pp. 303-14.

ings. In the following, the effect of propeller slipstream with consideration to axial supplementary velocity, slipstream rotation, and yaw on an airplane is described.

II. THE LIMITING CONDITIONS OF THE PROPELLER SLIPSTREAM DISTURBED BY AN AIRPLANE WING AND THEIR COMPLIANCE

1. General Form of Limiting Conditions for the Slipstream*

Visualize, for the present, the propeller slipstream replaced by a jet with constant jet velocity at infinity behind the propeller, without rotation or with constant twist, i.e., the flow to be a potential flow within and without the jet. The jet boundary is formed by a vorticity layer. Apart from this singular behavior of the flow at the jet boundary, there are other singularities in the flow; they are, particularly, the vortices formed by the vortex band behind the airplane wing. Now we shall attempt to define the flow within and without the jet. Aside from the singularities within these regions, the flow is determined by the behavior at the jet boundary.

At the jet boundary, the pressure of the outside flow (region I) must for reasons of equilibrium equal the pressure of the inside flow (region II). Hence the first limiting condition:

OUTSIDE INSIDE

$$p_I = p_{II} \qquad (1)$$

Otherwise, no stationary flow would be possible. The jet consists, in addition, always of the same particles of air; hence the jet boundary must consist of streamlines. The jet interferences are assumed to be small. Hence the jet may, although it is a little deformed, in first approximation be looked upon as round. The normal components of the velocities disappear at the jet boundary; then the following equations are applicable (fig. 1):

$$V_{n_I} = V_{r_I} \cos\gamma + V_{z_I} \sin\gamma = 0$$

$$V_{n_{II}} = V_{r_{II}} \cos\gamma + V_{z_{II}} \sin\gamma = 0$$

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*Sections 1 to 3, which are largely contained in Koning's report, are repeated here for the sake of clarity.

$V_{z_{I\,II}}$ TOTAL VELOCITY PARALLEL TO Z AXIS

$V_{r_{I\,II}}$ " " PERPENDICULAR TO Z AXIS

Here γ is the angle of the normal n to the jet surface and a plane perpendicular to the propeller slipstream axis (z.) This yields the second limiting condition at:

Axis.

$$\frac{v_{r_I}}{v_{z_I}} = \frac{v_{r_{II}}}{v_{z_{II}}} \quad \text{- Normal vel. Condition} \tag{2}$$

z should be perpendicular to jet boundary

2. Restatement of Limiting Conditions

Let $\overline{V}_I$ and $\overline{V}_{II}$ present the mean parallel velocity inside and outside of the jet, the deviations therefrom, that is, the interference velocities, induced by the wing, for instance, to have the components v_x, v_y, v_z. According to the pressure equation, it is: ?

Virtual slipstream direction — Actual slipstream direction

$$p_{I,II} + \frac{\rho}{2}\left[\left(\overline{V}_{I,II}\,x\cos\gamma + v_{z_{I,II}}\right)^2 + v_{y_{I,II}}^2 + v_{x_{I,II}}^2\right] = C_{I,II}$$

With p_o denoting the pressure at the point of vanishing interference velocity, it is:

$$C_{I,II} = p_{o_{I,II}} + \frac{\rho}{2}\left(\overline{V}^2_{I,II}\right)$$

If the interference velocities can be assumed to be so small that their squares can be neglected, the pressure equation becomes

$$p_{I,II} + \rho\,\overline{V}_{I,II}\,v_{z_{I,II}}\,x\cos\gamma = p_{o_{I,II}}$$

Since $v_{z_{I,II}}$ disappears at infinity and $p_I = p_{II}$ at the boundary, we have $p_{o_I} = p_{o_{II}} = p_o$. Owing to the pressure equality at the boundary, it follows that

$$\overline{V}_I\,v_{z_I} = \overline{V}_{II}\,v_{z_{II}} \quad \text{- Pressure condition}$$

$\overline{V}_I$ and $\overline{V}_{II}$ are the mean parallel velocities in direction of the propeller force, which in first approximation may be assumed to be coincident with the direction of the propeller axis. Let φ_I and φ_{II} indicate the potential of the interference flow. On account of

$$v_{z_I} = \frac{\partial \varphi_I}{\partial z} \quad ; \quad v_{z_{II}} = \frac{\partial \varphi_{II}}{\partial z}$$

the first limiting condition then takes the form

$$\overline{V}_I \frac{\partial \varphi_I}{\partial z} = \overline{V}_{II} \frac{\partial \varphi_{II}}{\partial z}$$

Since $\overline{V}_I$ and $\overline{V}_{II}$ are constant, the multiplication by dz followed by integration affords

$$\overline{V}_I \varphi_I = \overline{V}_{II} \varphi_{II} \qquad (3)$$

Pressure

for

$$\varphi_{I,II} = \int_{-\infty}^{z} \frac{\partial \varphi_{I,II}}{\partial z} dz + \varphi(-\infty)$$

The constant $\varphi(-\infty)$ has the same value in every point of the jet boundary at infinity. For the second limiting condition, we have:

$$V_{r/\text{boundary}} = \overline{V}_{r/R} + v_{r/R} = v_{r/R}; \quad V_{z/\text{boundary}} = \overline{V}_{z/R} + v_{z/R}$$

and

$$\frac{V_r}{V_z} = \frac{v_r}{\overline{\overline{V}}_z + v_z} = \frac{v_r}{\overline{\overline{V}}_z}\left(1 - \frac{v_z}{\overline{\overline{V}}} \pm \ldots\right) \sim \frac{v_r}{\overline{\overline{V}}_z}$$

Mean vel. axis slipstream

Mean vel. whole slipstream ?

On account of

$$v_{r_{I,II}} = \frac{\partial \varphi_{I,II}}{\partial r}$$

the second limiting condition reads herewith.

normal velocity condition

$$\overline{V}_{II}\frac{\partial\,\varphi_I}{\partial\,r} = \overline{V}_I\,\frac{\partial\,\varphi_{II}}{\partial\,r} \qquad (4)$$

Putting

$$\overline{V}_{II} = \overline{V}_I + s'\,\overline{V}_I, \quad \overline{V}_{II} = S\,\overline{V}_I$$

$S = \frac{\overline{V}_{II}}{V_I}$ (potential slipstream function)

that is,

$$S = 1 + s'$$

Dynamical slipstream direction see page 27.

the two limiting conditions assume the form

$$\varphi_I = S\,\varphi_{II} \qquad (5a)$$

pressure

$$S\,\frac{\partial\varphi_I}{\partial r} = \frac{\partial\,\varphi_{II}}{\partial\,r} \qquad (5b)$$

normal vel.

3. The Conditions Far Downstream from Wing and Propeller

In order to be able to study the effect of the slipstream on the airplane wing, it is recommended, for the sake of simplified conditions, to first define the flow far downstream from the wing and propeller with consideration to the limiting conditions. It may be presumed that the disturbances at that distance are of vanishing effect on the shape of the slipstream and of the vortex sheet induced by the wing, hence, specifically assumed that the interference component in the direction z disappears.

= exactly

Then the potential ~~is:~~ a solution of !

$$\Delta\,\varphi_{I,II} = \frac{\partial^2\varphi_{I,II}}{\partial\,x^2} + \frac{\partial^2\varphi_{I,II}}{\partial\,y^2} = 0 \qquad (6)$$

Laplace Equation

The disturbances at the jet boundary perpendicular to axis z are assumed to be small enough so that the slipstream may be figured as not being deformed, i.e., as being circular cylindrical. The radius of the circle is to be $R = 1$. In view of the potential reflection on a circle, the solution of the problem - to define $\varphi_{I,II}$ as a real part of a function $\chi_{I,II}(\underline{z})$ - is then comparatively simple. The following chapters therefore contain first the reflection of a vortex, a doublet, and a parallel flow on a circle in such a manner that it becomes the potential line.

4. Reflection of Vortex on the Circle*

We analyze the flow χ_a of a vortex of circulation $d\,\Gamma_a$ at a point $\underline{z}_a = x_a + iy_a = r_o\,e^{i\alpha_o}$ outside of the unit circle and the flow χ_i' of a vortex with the same circulation $d\Gamma_i$ in the reflection point $\underline{z}_i = x_i + iy_i = \frac{1}{\overline{\underline{z}_a}} = \frac{x_a + iy_a}{|\underline{z}_a|^2}$ on the unit circle (fig. 2). Then

$$\left.\begin{aligned} \chi_a &= -\,i\,\frac{d\,\Gamma_a}{2\,\pi}\,\ln\,(\underline{z} - \underline{z}_a) \\ \chi_i' &= -\,i\,\frac{d\,\Gamma_i}{2\,\pi}\,\ln\,(\underline{z} - \underline{z}_i) \end{aligned}\right\} \quad \cdots\cdots\cdots (7)$$

Furthermore, the flow χ_o induced by circulation $d\,\Gamma_o$ through a vortex located in $\underline{z} = 0$ is

$$\chi_o = -\,i\,\frac{d\,\Gamma_o}{2\,\pi}\,\ln\,\underline{z}$$

We find, if $\underline{z}$ passes through the points of the unit circle:

$$\underline{z}_a = r_o e^{i\,\alpha_o}; \qquad \underline{z}_i = \frac{1}{r_o}\,e^{i\,\alpha_o}$$

$$\underline{z} - \underline{z}_a = \cos\,\alpha - r_o\,\cos\,\alpha_o + i(\sin\,\alpha - r_o\,\sin\,\alpha_o);$$

$$\underline{z} - \underline{z}_i = \cos\,\alpha - \frac{1}{r_o}\,\cos\,\alpha_o + i\left(\sin\,\alpha - \frac{1}{r_o}\,\sin\,\alpha_o\right);$$

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*In contrast to Koning's method of presentation, we employed the functions of complex variables, because they make the result easier to obtain.

$$\underline{z} - \underline{z}_a = r' e^{i \arctan \frac{\sin \alpha - r_o \sin \alpha_o}{\cos \alpha - r_o \cos \alpha_o}}$$

$$\underline{z} - \underline{z}_i = r'' e^{i \arctan \frac{\sin \alpha - \frac{1}{r_o} \sin \alpha_o}{\cos \alpha - \frac{1}{r_o} \cos \alpha_o}}$$

$$r' = \sqrt{1 + r_o^2 - 2\, r_o \cos (\alpha - \alpha_o)}$$

$$r'' = \sqrt{1 + \frac{1}{r_o^2} - 2 \frac{1}{r_o} \cos (\alpha - \alpha_o)}$$

With

$$\ln (\underline{z} - \underline{z}_a) = \ln r' + i \arctan \frac{\sin \alpha - y_a}{\cos \alpha - x_a}$$

$$\ln (\underline{z} - \underline{z}_i) = \ln r'' + i \arctan \frac{\sin \alpha - y_i}{\cos \alpha - x_i}$$

the potential and stream function of the individual vortices on the unit circle are:

$$\left.\begin{aligned}
\varphi_a &= \frac{d\, \Gamma_a}{2\, \pi} \arctan \frac{\sin \alpha - y_a}{\cos \alpha - x_a} \\
\varphi_i' &= \frac{d\, \Gamma_i}{2\, \pi} \arctan \frac{\sin \alpha - \frac{y_a}{|\underline{z}_a|^2}}{\cos \alpha - \frac{y_a}{|\underline{z}_a|^2}} \\
\varphi_o &= \frac{d\, \Gamma_o}{2\, \pi} \alpha
\end{aligned}\right\} \qquad (8a)$$

$$\left.\begin{aligned}
\psi_a &= -\frac{d\,\Gamma_a}{2\,\pi}\,\frac{1}{2}\,\ln\left[1 + x_a{}^2 + y_a{}^2 - 2\,x_a\cos\alpha - 2\,y_a\sin\alpha\right] \\
\psi_i{}' &= -\frac{d\,\Gamma_i}{2\,\pi}\left[\ln\frac{1}{r_o} + \frac{1}{2}\ln(1+x_a{}^2+y_a{}^2 - 2\,x_a\cos\alpha - 2\,y_a\sin\alpha)\right] \\
\psi_o &= 0
\end{aligned}\right\}(8b)$$

If r is the amount of any point $\underline{z}$, then $\dfrac{\partial\varphi}{\partial r} = \dfrac{\partial\psi}{r\,\partial\alpha}$. On the unit circle, $r = R = 1$, hence the normal components of the velocity at that point read:

$\dfrac{\partial\psi_a}{r\,\partial\alpha} =$

equal if $d\Gamma_i = d\Gamma_a$

$$\left.\begin{aligned}
\frac{\partial\varphi_a}{\partial n} &= -\frac{d\,\Gamma_a}{2\,\pi}\,\frac{x_a\sin\alpha - y_a\cos\alpha}{1 + x_a{}^2 + y_a{}^2 - 2\,x_a\cos\alpha - 2\,y_a\sin\alpha} \\
\frac{\partial\varphi_i{}'}{\partial n} &= -\frac{d\,\Gamma_i}{2\,\pi}\,\frac{x_a\sin\alpha - y_a\cos\alpha}{1 + x_a{}^2 + y_a{}^2 - 2\,x_a\cos\alpha - 2\,y_a\sin\alpha} \\
\frac{\partial\varphi_o}{\partial n} &= 0
\end{aligned}\right\}\ldots(9)$$

with $\dfrac{\partial\varphi}{\partial n}$ written for $\dfrac{\partial\varphi}{\partial r}$. The normal components of identically great equivalent vortices in reflected points are identically great and in identical direction on the reflection circle. For

$$\underline{z} = e^{i\alpha} = e^{i(\alpha_o + \beta)}$$

it further affords with $d\,\Gamma_a = d\,\Gamma_i = d\,\Gamma$

where $\beta = \alpha - \alpha_o$

$$\chi_a + \chi_i' = -i \frac{d\Gamma}{2\pi} \ln (\underline{z} - \underline{z_a})(\underline{z} - \underline{z_i})$$

$$= -i \frac{d\Gamma}{2\pi} \ln \left[e^{i(2\alpha_o - 2\beta)} - \left(r_o + \frac{1}{r_o}\right) e^{i(2\alpha_o + \beta)} + e^{i\ 2\alpha_o} \right]$$

$$= -i \frac{d\Gamma}{2\pi} \ln \left[e^{i\ 2\alpha_o} \left(e^{i-2\beta} - \left(r_o + \frac{1}{r_o}\right) e^{i\beta} + 1 \right) \right]$$

$$= -i \frac{d\Gamma}{2\pi} \ln \left[(\cos 2\alpha_o + i \sin 2\alpha_o) \right.$$

$$\left. \left[\cos 2\beta + i \sin 2\beta - \left(r_o + \frac{1}{r_o}\right)(\cos \beta + i \sin \beta) + 1 \right] \right]$$

which, however, because $\cos 2\beta + 1 = 2\cos^2 \beta$ and $\sin 2\beta = 2 \sin \beta \cos \beta$, changes to

$$\chi_a + \chi_i' = -i \frac{d\Gamma}{2\pi} \times$$

$$\times \left[i\ 2\alpha_o + \ln r''' e^{i \arctan \frac{2 \sin \beta \cos \beta - \left(r_o + \frac{1}{r_o}\right) \sin \beta}{2\cos^2 \beta - \left(r_o + \frac{1}{r_o}\right) \cos \beta}} \right]$$

This gives for the potential

$$\varphi_a + \varphi_i' = \frac{d\Gamma}{2\pi} \left[2\alpha_o + \arctan \frac{\sin \beta \left[2 \cos \beta - \left(r_o + \frac{1}{r_o}\right) \right]}{\cos \beta \left[2 \cos \beta - \left(r_o + \frac{1}{r_o}\right) \right]} \right]$$

$$= \frac{d\Gamma}{2\pi} \left[2\alpha_o + \arctan \tan \beta \right]$$

$$= \frac{d\Gamma}{2\pi} (2\alpha_o + \beta) = \frac{d\Gamma}{2\pi} (2\alpha_o + \alpha - \alpha_o)$$

and

$$\varphi_a + \varphi_i' = \frac{d\Gamma}{2\pi} (\alpha + \alpha_o) \quad \ldots \ldots \ldots \quad (10)$$

$$\varphi_o = \frac{d\Gamma_o}{2\pi} \alpha_o$$

Hence $\varphi_a + \varphi_i' = \varphi_0$ ~~up to~~ except for one constant. When reflecting a vortex on a circle through an identically great and equivalent vortex, the potential $\varphi_a + \varphi_i' + \varphi_0$ on the circle, in whose center an identically great contrary vortex $d\,\Gamma_0 = -d\,\Gamma_i = -d\,\Gamma_a$ is placed, becomes constant, that is, the reflecting circle becomes a potential line. As reflection of a vortex on the outside a vortex in the reflection point and in the center and vice versa must be considered. In other words, if χ_i is to be the reflection of flow χ_a, then χ_i must be

$$\chi_i = \chi_i' + \chi_0$$

Hence

$$\varphi_a + \varphi_i' = \frac{d\Gamma}{2\pi}(\alpha + \alpha_0)$$

$$\varphi_0 = -\frac{d\Gamma}{2\pi}\alpha_0 \quad \text{if } d\Gamma = -d\Gamma_0$$

$$\varphi_a + \varphi_i' + \varphi_0 = \frac{d\Gamma}{2\pi}\alpha_0 = \text{const}$$

$$\varphi_a + \varphi_i = \text{const.}$$

$$\frac{\partial \varphi_a}{\partial n} = -\frac{\partial \varphi_i}{\partial n}$$

$$\frac{\partial \varphi_a}{\partial n} = \frac{\partial \varphi_i'}{\partial n} \quad \text{equation 9}$$

5. Reflection of Vortex Doublet on a Circle*

The vortex band in the wake of a wing can also be visualized as being due to a doublet. The density $\frac{d\,\lambda}{d\,s}$ of this superposed doublet is equal to the circulation Γ existing in the accompanying part of the wing. We shall analyze the reflection of a doublet on the circle for which the normal components of the velocity of the inside and outside doublet on the reflecting circle become equal and the potential is constant, as in the case of the single vortex.

Since the vortices forming the doublet are identically great and contrary, the reflection of a doublet located at finity requires no additional doublet. The outer doublet is to have the moment $d\,\lambda_a$ and an axial direction differing from axis y by an angle ω_a, and located at

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*The reflection of the doublets practiced here, is not contained in Koning's report. But it proves itself advantageous and indicates the way to the propeller setting not treated by Koning, as shown in fig. 6.

$\underline{z}_a = r_o\, e^{i\,\alpha_o}$ (fig. 3). The inner doublet is to have the moment $d\,\lambda_i$ and an axial direction differing by angle ω_i from the axis y, and located in the reflection point to $\underline{z}_a$, that is, at $\underline{z}_i = \frac{1}{r_o}\, e^{i\,\alpha_o}$. The unit circle again serves as reflection circle. The flow potentials of both vortices on the unit circle are:

$$\left.\begin{aligned}
\chi_a &= -\,i\,d\,\lambda_a\, e^{i\omega_a}\, \frac{1}{\underline{z} - \underline{z}_a} \\
&= -\,i\,d\,\lambda_a\, e^{i\omega_a}\, \frac{1}{e^{i(\alpha_o + \beta)} - r_o\, e^{i\,\alpha_o}} \\
&= -\,i\,d\,\lambda_a\, e^{i(\omega_a - \alpha_o)}\, \frac{1}{e^{i\beta} - r_o} \\
\chi_i &= -\,i\,d\,\lambda_i\, e^{i\omega_i}\, \frac{1}{\underline{z} - \underline{z}_i} \\
&= -\,i\,d\,\lambda_i\, e^{i\omega_i}\, \frac{1}{e^{i(\alpha_o + \beta)} - \frac{1}{r_o}\, e^{i\,\alpha_o}} \\
&= -\,i\,d\,\lambda_i\, e^{i(\omega_i - \alpha_o)}\, \frac{1}{e^{i\,\beta} - \frac{1}{r_o}}
\end{aligned}\right\} \quad \ldots .(11)$$

The potentials therefore are:

$$\varphi_a = +\,d\,\lambda_a \left[\cos\,(\omega_a - \alpha_o)\, \frac{-\sin\,\beta}{r_o^2 + 1 - 2\,r_o\,\cos\,\beta} + \sin\,(\omega_a - \alpha_o)\, \frac{\cos\,\beta - r_o}{r_o^2 + 1 - 2\,r_o\,\cos\,\beta} \right]$$

$$\varphi_i = +\,d\,\lambda_i \left[\cos\,(\omega_i - \alpha_o)\, \frac{-\sin\,\beta}{\frac{1}{r_o^2} + 1 - 2\,\frac{1}{r_o}\,\cos\,\beta} + \sin\,(\omega_i - \alpha_o)\, \frac{\cos\,\beta - \frac{1}{r_o}}{\frac{1}{r_o^2} + 1 - 2\,\frac{1}{r_o}\,\cos\,\beta} \right]$$

To insure $\varphi_a + \varphi_i = \text{const.}$ on the unit circle, it is necessary to write

$$d\,\lambda_i = -\,d\,\lambda_a \frac{1}{r_o^2};\ (\omega_i - \alpha_o) = -\,(\omega_a - \alpha_o) \tag{12}$$

$\omega_i - \alpha_o = \vartheta_i$ and $\omega_a - \alpha_o = \vartheta_a$ are the angles between the axes of the double vortices and the perpendiculars to the radii toward the doublets (fig. 4). The result is

$$\left.\begin{aligned}
\varphi_a &= +\,d\,\lambda_a \left[\cos\,\vartheta_a \frac{-\sin\beta}{r_o^2 + 1 - 2\,r_o\cos\beta} + \sin\,\vartheta_a \frac{\cos\beta - r_o}{r_o^2 + 1 - 2\,r_o\cos\beta}\right] \\
\varphi_i &= -\,d\,\lambda_a \left[\cos\,\vartheta_a \frac{-\sin\beta}{r_o^2 + 1 - 2\,r_o\cos\beta} - \sin\,\vartheta_a \frac{\cos\beta - \frac{1}{r_o}}{r_o^2 + 1 - 2\,r_o\cos\beta}\right]
\end{aligned}\right\} \tag{13}$$

and consequently:

$$\varphi_a + \varphi_i = -\,d\,\lambda_a \frac{\sin\,\vartheta_a}{r_o} = \text{const.} \tag{14}$$

Thus $\varphi_a = -\,\varphi_i$ on the unit circle up to the constant at the right. The stream function on the unit circle is

$$\left.\begin{aligned}
\psi_a &= -\,d\,\lambda_a \left[\sin\,\vartheta_a \frac{\sin\beta}{r_o^2 + 1 - 2\,r_o\cos\beta} + \cos\,\vartheta_a \frac{\cos\beta - r_o}{r_o^2 + 1 - 2\,r_o\cos\beta}\right] \\
\psi_i &= +\,d\,\lambda_a \left[-\sin\,\vartheta_a \frac{\sin\beta}{r_o^2 + 1 - 2\,r_o\cos\beta} + \cos\,\vartheta_a \frac{\cos\beta - \frac{1}{r_o}}{r_o^2 + 1 - 2\,r_o\cos\beta}\right]
\end{aligned}\right\} \tag{15}$$

and, after subtraction:

$$\psi_a - \psi_i = + d\lambda_a \frac{\cos \vartheta_a}{r_o} = \text{const.} \tag{16}$$

and as far as the constant $\psi_a = \psi_i$ on the right. Because $\frac{\partial \varphi}{\partial r} = \frac{\partial \psi}{r \partial \alpha}$, we have on the unit circle, with n indicating the direction of the normals:

$$\frac{\partial \psi_a}{r \partial \alpha} = \frac{\partial \psi_i}{r \partial \alpha} \quad \ldots\ldots\ldots\ldots (17a)$$

$$\frac{\partial \varphi_a}{\partial n} = \frac{\partial \varphi_i}{\partial n} \quad \ldots\ldots\ldots\ldots (17b)$$

Equation (17a) follows from equation (16), (17b) from (17a). In contrast, equation (14) gives the resultant of the interference velocity, because

$$\frac{\partial}{\partial n} (\varphi_a + \varphi_i) = v_n$$

If the doublet distribution extends over an element $d\,\underline{z}_a$ starting from $\underline{z}_a$ and the axis perpendicular to it, the reflected doublet is distributed over an element $d\,\underline{z}_i$. Suppose ds_a and ds_i, that is, $|d\underline{z}_a| = ds_a$ and $|d\underline{z}_i| = ds_i$ are the lengths of these elements. Then the moments of the vortex distributions are $d\lambda_a = \Gamma_a ds_a$ and $d\lambda_i = \Gamma_i ds_i$, and

$$\Gamma_i ds_i = - \frac{1}{r_o^2} \Gamma_a ds_a \quad \ldots\ldots\ldots\ldots (18)$$

according to equation (12). But with $\underline{z}_i = \frac{1}{\underline{z}_a} = \frac{z_a}{r_o^2}$, it is

$$d\,\underline{z}_i = \frac{dx_a}{r_o^2} - \frac{2\,x_a}{r_o^3}\, d\,r_o + i\left(\frac{d\,y_a}{r_o^2} - \frac{2\,y_a}{r_o^3}\, d\,r_o\right)$$

and

$$\left| d\, \underline{z}_i \right|^2 = \frac{1}{r_o^4} \left(dx_a^2 + dy_a^2 \right) + \frac{4}{r_o^6} \left(x_a^2 + y_a^2 \right) d\, r_o^2$$

$$- \frac{4}{r_o^5} \left(x_a\, dx_a + y_a\, dy_a \right) d\, r_o$$

$$= \frac{1}{r_o^4}\, d\, s_a^2 + \frac{4}{r_o^4}\, d\, r_o^2 - \frac{4}{r_o^4}\, d\, r_o^2 = \frac{1}{r_o^4}\, d\, s_a^2$$

Hence

$$d\, s_i = \frac{1}{r_o^2}\, d\, s_a$$

and equation (18) becomes

$$\Gamma_i = - \Gamma_a \quad \text{. . . . . (19)}$$

6. Reflection of Parallel Flow on a Circle

If the direction of the slipstream does not coincide with the direction of flight, the velocity of the outside flow has a component transverse to the slipstream. If this is small, the assumptions for the derivation of the limiting conditions are applicable. Suppose the parallel flow on the outside is:

$$\chi_a = i\, v_y \cdot \underline{z} \quad \text{. . . . . (20)}$$

Then, putting $\frac{\partial \varphi}{\partial s} = v_s$, the y axis is opposite to the transverse flow component, and

$$\varphi_a = - v_y\, y; \qquad \psi_a = v_y\, x$$

The reflection is, as is known, a double vortex source with axes direction parallel to the transverse flow direction if the circle is to be a potential line. The complex potential for it reads

$$\chi_i = i\, v_y \frac{1}{\underline{z}} \quad \text{. . . . . (21)}$$

With $x = r \cos \alpha$ and $y = r \sin \alpha$ and $z = r\, e^{i\alpha}$, we have

$$\left.\begin{array}{ll} \varphi_a = - v_y\ r \sin\alpha & \psi_a = v_y\ r\cos\alpha \\ \varphi_i = v_y \dfrac{\sin\alpha}{r} & \psi_i = v_y \dfrac{\cos\alpha}{r} \end{array}\right\} \quad . . . . (22)$$

Hence the combined flow on the unit circle $(r = 1)$ becomes:

$$\left.\begin{array}{l} \varphi_a + \varphi_i = 0 \\ \psi_a - \psi_i = 0 \end{array}\right\} \quad . . . . (23)$$

From equations (22) and (23) follows:

$$\frac{\partial\ \varphi_a}{\partial\ n} = \frac{\partial\ \varphi_i}{\partial\ n} = - v_y \sin\alpha \quad . . . . (24)$$

a result which can equally be obtained direct.

7. The Supplementary Flow Created by the Coordination Of Vortices or Doublets with the Slipstream*

Suppose that φ_{A_a} is the potential of the initial interference flow, caused by the single or double vortices located outside of the slipstream, and φ_{B_i} that caused by those located on the inside. As the limiting conditions cannot be satisfied with these potentials, the existing singularities are for the present reflected on the assumedly circular jet boundary. The potentials created by the reflections are φ_{A_i} and φ_{B_a}. Then, if the circle is posed as potential line, it is on the circle up to constants

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*The treatment of the combined flow quotas here is somewhat more general than Koning's which seemed advisable on account of the dissimilarity of the vortex doublets and the parallel flow introduced as elements of disturbance aside from the single vortices.

$$\left.\begin{aligned} \varphi_{A_i} &= -\varphi_{A_a} + \text{const} \qquad & \frac{\partial \varphi_{A_i}}{\partial n} &= \frac{\partial \varphi_{A_a}}{\partial n} \\ \varphi_{B_i} &= -\varphi_{B_a} + \text{const} \qquad & \frac{\partial \varphi_{B_i}}{\partial n} &= \frac{\partial \varphi_{B_a}}{\partial n} \end{aligned}\right\} \quad \cdots \cdots (25)$$

The ultimate solution satisfying the limiting condition is written in the form

$$\left.\begin{aligned} \varphi_I &= \varphi_{AI} + \varphi_{BI} \quad \text{(outside)} \\ \varphi_{II} &= \varphi_{AII} + \varphi_{BII} \quad \text{(inside)} \end{aligned}\right\} \quad \cdots \cdots (26)$$

whereby the summands on the right-hand side require further treatment. The physically proven fact of the limiting conditions indicates that only the singularities contained in φ_{A_a} are present. In consequence, φ_I is built up only from the potentials of the outside singularities, their reflections, the inside singularities and certain terms necessary to satisfy the limiting conditions. The reflections of the inside singularities are disregarded. The supplementary terms can only be proportions of the already existing potential, and specifically, they can only be formed with the singularities located on the inside, because the potential is definitely prescribed to one part through the externally existing singularities. For the inside flow the corresponding considerations are applicable. With n as proportional factor, we write:

$$\left.\begin{aligned} \varphi_{AI} &= \varphi_{A_a} + n_{A_i}\varphi_{A_i}; \qquad & \varphi_{BI} &= \varphi_{B_i} + n_{B_i}\varphi_{B_i} \\ \varphi_{AII} &= \varphi_{A_a} + n_{A_a}\varphi_{A_a}; \qquad & \varphi_{BII} &= \varphi_{B_i} + n_{B_a}\varphi_{B_a} \end{aligned}\right\} \quad (27)$$

Since φ_{A_a} and φ_{B_i} are unrelated, the limiting conditions must be satisfied for every part φ_A and φ_B. Equations (5a) and (5b) therefore split into four equations:

$$\varphi_{AI} = S\,\varphi_{AII}; \qquad \varphi_{BI} = S\,\varphi_{BII};$$

$$S\,\frac{\partial \varphi_{AI}}{\partial n} = \frac{\partial \varphi_{AII}}{\partial n}; \qquad S\,\frac{\partial \varphi_{BI}}{\partial n} = \frac{\partial \varphi_{BII}}{\partial n}$$

With equations (25) and (27), it then affords

$$1 - n_{A_i} = S(1 + n_{A_a}); \qquad 1 + n_{B_i} = S(1 - n_{B_a})$$

$$S(1 + n_{A_i}) = (1 + n_{A_a}); \qquad S(1 + n_{B_i}) = (1 + n_{B_a})$$

The solution gives

$$n_{A_a} = -\frac{(S-1)^2}{S^2+1}; \qquad n_{B_a} = \frac{S^2-1}{S^2+1}$$

$$n_{A_i} = -\frac{S^2-1}{S^2+1}; \qquad n_{B_i} = -\frac{(S-1)^2}{S^2+1}$$

Assuming now that s' is so small that squared terms and terms of higher order can be neglected, it is

$$(S-1)^2 = s'^2 \sim 0$$

$$S^2 + 1 = 2 + 2\,s' + s'^2 \sim 2(1 + s')$$

$$\frac{(S-1)^2}{S^2+1} \sim 0$$

$$S^2 - 1 = 2\,s' + s'^2 \sim 2\,s'$$

$$\frac{S^2-1}{S^2+1} = s' - s'^2 \pm \ldots \sim s'$$

The factors of the supplementary potential finally are:

$$\left.\begin{aligned} n_{A_a} &= 0; & n_{B_a} &= s' \\ n_{A_i} &= -s'; & n_{B_i} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots (28)$$

and the solution satisfying the limiting conditions reads:

$$\left.\begin{aligned} \varphi_{I} &= \varphi_{A_a} - s'\,\varphi_{A_i} + \varphi_{B_i} \\ \varphi_{II} &= \varphi_{A_a} + \varphi_{B_i} + s'\,\varphi_{B_a} \end{aligned}\right\} \quad \ldots\ldots\ldots (29)$$

8. Reliability and Practicability of Introducing (Vortex) Doublets*

Although this reflection of doublets may seem at first to be something superfluous, it is found that it in fact clarifies the determination of the supplementary flow due to the interaction of wing and propeller slipstream. The outline of a wing with dihedral on the circular boundary of a propeller slipstream is illustrated in figure 5. Let the lift distribution be elliptic.

The the flow outside the slipstream is presented:

1) By the flow A_a, i.e., the distribution of doublets outside the propeller slipstream corresponding to the circulation distribution Γ_o (fig. 5, A_a);

2) By the flow B_i, i.e., the distribution of the doublets inside the propeller slipstream corresponding to Γ_o (fig. 5, B_i);

3) By the flow A_i, i.e., the s' times reflection of the external distribution of doublets on the circular boundary of the propeller slipstream (fig. 5, A_i).

The flow on the inside of the propeller slipstream is presented:

1) By the flow A_a, i.e., the distribution of doublets outside the slipstream corresponding to Γ_o (fig. 5, A_a);

2) By the flow B_i, i.e., the distribution of the doublets inside the slipstream corresponding to Γ_o (fig. 5, B_i);

3) By the flow B_a, i.e., the s' times reflection of the inner distribution of doublets inside the slipstream (fig. 5, B_a).

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*Koning, omitting the concept of doublets found it difficult to satisfy the boundary conditions for the airfoil spanning the jet, and was compelled to treat it differently.

Since the circulation distribution affords the vortex distribution of the vortex sheet in the wake of the wing by differentiation, the vortex distributions corresponding to the enumerated proportions of the flow may also be presented. The presented distributions of doublets of these proportions all terminate, however, at the slipstream boundary with a finite value Γ_0 and $\pm s\ \Gamma_0$, respectively. To these finite values of the distribution of doublets correspond one isolated vortex $\pm\Gamma_0$ and $\pm s\ \Gamma_0$, each, respectively, of the vortex distributions. The vortex distributions correspond to the enumerated flow proportions are illustrated in figure 6.

For qualitative studies of the combined action of a wing and a propeller slipstream, it obviously suffices to disregard the span effects, the chord distribution, and the twist, leaving a non-twisted wing of constant chord and infinite span in the propeller slipstream. In the absence of slipstream the spanwise circulation distribution of this wing would be constant. The corresponding distribution of doublets with slipstream are shown in figure 7. The resultant vortex distribution is contained in figure 8. It consists of one single vortex each at the point of penetration of wing into the slipstream.

This result could not have been obtained directly when proceeding from the vortex distribution corresponding to the circulation distribution without slipstream. Only by visualizing the lift distribution at the point of penetration of the wings with the slipstream interrupted for a very small piece can the above result be obtained in correspondence with the single vortices, which then would have to be visualized as shedding inside and outside from the point of interruption. Since they have opposite sense of rotation inside and outside, possess finite strength, and are closely adjacent, they would cancel in the absence of slipstream, hence it is admissible to visualize the wing as being interrupted at the place of penetration. But, that this must be done in order to be able to satisfy the limiting conditions with the help of the reflection method, is only ascertainable in the roundabout way of the distribution of doublets.

9. Yawed Slipstream*

A slipstream in yaw undergoes a change in direction. The original direction, substantially coincident with the propeller axis, is termed the virtual slipstream direction. The potential of the interference share, of the flow arriving obliquely at the virtual slipstream direction is φ_{A_a}, the velocity component parallel to the virtual slipstream direction is $\overline{V}_I$. This notation $\overline{V}_I$ is legitimate and indicates the velocity introduced elsewhere; for, as seen shortly, the virtual jet direction coincides with the direction of the propeller force.

Locating the coordinate system in the plane perpendicular to the virtual slipstream direction as before, the potential of the external initial interference flow is

$$\varphi_{A_a} = - v_{y_{virt}} y = - v_{y_{virt}} r \sin \alpha$$

The pertinent potential of the reflection is

$$\varphi_{A_i} = \frac{v_{y_{virt}} \sin \alpha}{r}$$

No other singularities being present, equations (29) read:

$$\left.\begin{aligned} \varphi_I &= \varphi_{A_a} - s' \varphi_{A_i} = -v_{y_{virt}} r \sin \alpha - s' v_{y_{virt}} \frac{\sin \alpha}{r} \\ &= -\left(1 + \frac{s'}{r^2}\right) y \; v_{y_{virt}} \\ \varphi_{II} &= \varphi_{A_a} = -y \; v_{y_{virt}} \end{aligned}\right\} \quad (30)$$

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*The analysis of Koning's limiting conditions summarily discloses that the supplementary flow due to a yawed slipstream imbedded in the outside flow must be as if the slipstream were frozen in a solid cylinder. Because this flow satisfies summarily the continuity equation (2) and the conditions equations (1) and (3), respectively. But, suspecting difficulties with the effect of the yawed flow, the derivation was here carried out exactly, in conformity with the general results.

Now, some predictions can be made regarding the processes within and without the slipstream. The axial velocity within the virtual propeller slipstream is $\overline{V}_{II}$. It is superposed by the interference velocity $v_{y_{virt}}$ perpendicular to it, as a result of the potential φ_{II} (fig. 9). Both velocities, vectorially added, give the resultant velocity $\underline{v}_{res}$ and with it the actual direction of the propeller slipstream.

If $\underline{v}_{zus}$ is the change in flying speed $\underline{v}_o$ with respect to $\underline{v}_{res}$, that is, if

$$\underline{v}_{res} = \underline{v}_o - \underline{v}_{zus}$$

it is seen that the direction of the propeller force must fall in the direction of $\underline{v}_{zus}$, because, according to the momentum theory, the acting force is proportional to the change in speed. Since $\underline{v}_{zus}$ is parallel to the virtual jet direction, the direction of the propeller force coincides with the virtual propeller slipstream direction. If the direction of the propeller force, that is, in first approximation, the propeller axis, forms the angle ν with the flight direction, then

$$v_{y_{virt}} = v_o \sin \nu$$

If ν is small, then $v_{res} \sim v_o + v_{zus}$. The actual jet slopes at $\alpha_{i_{Str}} = \nu - \nu'$ toward the flight direction. Since

$$\nu = \frac{v_{y_{virt}}}{v_o}; \qquad \nu' = \frac{v_{y_{virt}}}{v_o + v_{zus}}$$

we have

$$\alpha_{i_{Str}} = \frac{v_{y_{virt}}}{v_o} - \frac{v_{y_{virt}}}{v_o + v_{zus}} = \frac{v_{y_{virt}}}{v_o} \frac{v_{zus}}{(v_o + v_{zus})}$$

Then, however,

$$\underline{v}_{zus} = \underline{\overline{V}}_{II} - \underline{\overline{V}}_I = \underline{\overline{V}}_I + s'\underline{\overline{V}}_I - \underline{\overline{V}}_I = s'\underline{\overline{V}}$$

and

$$\frac{v_{zus}}{\overline{V}_I} = s'$$

With $\overline{V}_I = v_o \cos \nu$ and $s = s' \cos \nu$ the actual direction of the propeller slipstream then slopes at angle

$$\alpha_{i_{Str}} = \nu \frac{s\ v_o}{v_o + s\ v_o} \sim s\ \nu \tag{31}$$

toward the direction of flight. This result holds far behind wing and propeller for a particle lying on the inside of the jet. Outside of the jet, the velocity $\left(1 + \frac{s'}{r^2}\right) v_{y_{virt}}$ is superposed because φ_I is perpendicular to $\overline{V}_I$. Then, according to figure 10:

$$\tan\left(-\alpha_{i_a} + \nu\right) = \frac{\left(1 + \frac{s'}{r^2}\right) v_{y_{virt}}}{\overline{V}_I}$$

If the angles are small, we have

$$-\alpha_{i_a} + \nu = \left(1 + \frac{s}{r^2}\right) \frac{v_{y_{virt}}}{v_o}$$

and

$$-\alpha_{i_a} = -\frac{v_{y_{virt}}}{v_o} + \left(1 + \frac{s}{r^2}\right) \frac{v_{y_{virt}}}{v_o} = \frac{s}{r^2}\nu$$

For the slope of the particles in the outside zone toward the flight direction, it gives

$$\alpha_{i_a} = -\frac{s}{r^2}\nu \tag{32}$$

the propeller radius serving as length unit.

10. Effect of Jet Rotation*

It is assumed that the axial supplementary velocities in the jet are accompanied by a constant twist

tangential velocity

$$r\ v_u = \text{const}$$

Let $v_u = 0$ on the outside. Then the pressure condition is summarily complied with, because the pressure in the jet regulates itself accordingly. Since the radial components disappear on the inside as on the boundary and the outside flow itself is without radial components, the second limiting condition is also satisfied. The jet rotation has therefore no effect on the behavior at the jet boundary. This compliance with the limiting conditions can be proved equally well with equation (29). For the case in point, it is:

$$\left.\begin{aligned} \varphi_I &= \varphi_{B_i} \\ \varphi_{II} &= \varphi_{B_i} + s'\,\varphi_{B_a} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (33)$$

According to the assumptions φ_I must be zero or constant. Since $r\ v_u$ is to be constant, it must be

$\dfrac{\partial \varphi_{II}}{\partial \alpha} = \text{const}$ on the inside. As the jet boundary represents a surface of discontinuity, φ_{B_i} contains a component due to the potential of a vortex distribution. The vortices are uniformly distributed over the jet boundary with constant vortex density

$\Gamma' = \dfrac{d\,\Gamma}{d\,\sigma}$. If $\Gamma' d\sigma$ is the circulation of a single vortex distributed over the circumferential element $d\sigma$, the complex potential of the total vortex distrbution reads:

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*Concerning the jet rotation, it is also obvious that it does not disturb the limiting conditions, hence has no supplementary flows. The slipstream rotation is treated as rotation with constant twist, which corresponds more closely to the behavior of the propellers than Koning's rotation with constant angular velocity. But in view of originally suspected difficulties with jet rotation also, the discussed results were again used for an exact analysis of the effect of slipstream rotation.

$$\chi(\underline{z}) = -i\frac{\Gamma'}{2\pi}\oint \ln(\underline{z} - Z)d\sigma$$

where $\underline{z}$ is the starting point and Z the current point along the circle. With $Z = R\,e^{i\alpha}$ and $d\sigma = Rd\alpha$, we have $d\sigma = -iR\frac{dZ}{Z}$ and consequently

$$\chi(\underline{z}) = -\frac{\Gamma'}{2\pi}R\oint \ln(\underline{z} - Z)\frac{dZ}{Z}$$

Both cases $\left|\underline{z}\right| \gtrless \left|Z\right|$ are analyzed. If $\left|\underline{z}\right| > \left|Z\right|$, it is

$$\chi(\underline{z}_a) = -\frac{\Gamma'}{2\pi}R\left[\oint \ln\left(1 - \frac{Z}{\underline{z}}\right)\frac{dZ}{Z} + \oint \ln \underline{z}\frac{dZ}{Z}\right]$$

The first integral disappears, as can be proved by series expansion, leaving

$$\chi(\underline{z}_a) = -\frac{\Gamma'}{2\pi}R\cdot 2\pi\cdot i\cdot \ln \underline{z}$$

But, since $\Gamma = 2R\pi\Gamma'$ represents the total circulation, it is:

$$\chi(\underline{z}_a) = -i\frac{\Gamma}{2\pi}\ln \underline{z}$$

On the outside, the vortex distribution therefore acts like a vortex placed in the origin and whose circulation is equal to the total circulation of the vortex distribution. As φ_{B_i} is to disappear outside the circle, this may be achieved with an oppositely rotating vortex of the same circulation strength placed in the origin. Of course, it then remains to be proved whether this assumption corresponds to the conditions stipulated on the inside. If $\left|\underline{z}\right| > \left|Z\right|$, then

$$\chi(\underline{z}_i) = -\frac{\Gamma'}{2\pi}R\left[\oint \ln\left(1 - \frac{\underline{z}}{Z}\right)\frac{dZ}{Z} + \oint \ln Z\frac{dZ}{Z} + \pi i\oint\frac{dZ}{Z}\right]$$

The first integral disappears, leaving

$$\chi(\underline{z}_i) = -\frac{\Gamma'}{2\pi}R\left[2\pi\ i\ \ln R - 2\pi^2 + \pi\ i\ 2\pi\ i\right] = \Gamma - i\ \frac{\Gamma}{2\pi}\ \ln R$$

On the inside the potential of the vortex distribution is constant, thus contributing nothing to the velocity. For the vortex placed in the origin, it is:

$$\chi_o = +i\frac{\Gamma}{2\pi}\ln \underline{z}$$

hence

$$\frac{d\chi_o}{d\underline{z}} = \overline{\underline{w}} = u - i\ v = i\ \frac{\Gamma}{2\pi}\frac{1}{\underline{z}} = i\ \frac{\Gamma}{2\pi}\left(\frac{x}{r^2} - i\ \frac{y}{r^2}\right)$$

with $r^2 = x^2 + y^2$. As a result we have

$$u = \frac{\Gamma}{2\pi}\frac{y}{r^2}; \qquad v = -\frac{\Gamma}{2\pi}\frac{x}{r^2}$$

and

$$w = \frac{\Gamma}{2\pi}\frac{1}{r}$$

Now it shall be proved that w is the amount of the circumferential velocity v_u. From χ_o follows $\varphi = -\frac{\Gamma}{2\pi}\alpha$; hence $v_u = \frac{\partial\varphi}{\partial s} = \frac{\partial\varphi}{r\,\partial\alpha} = -\frac{\Gamma}{2\pi}\frac{1}{r}$. This would satisfy the condition $r\ v_u = \text{const}$; it is satisfied through the potential of the vortex distribution and the oppositely rotating vortex. There remains to be shown that φ_{B_a} on the inside is constant or disappears. φ_{B_a} is the potential of the reflected singularities located on the inside. The reflection of the single vortex shifts to infinity, its potential assumes a constant value at finity, hence does not contribute to the velocity inside of the circle. The reflection of the vortex distribution is the vortex distribution itself, which on the inside has no effect on the velocity. The velocity on the inside is therefore simply caused by the single vortex. In view of the physically cited fact of vortex distribution, there is no velocity at the boundary on the outside. With it, however, the conditions are the same as assumed at the beginning of the section.

III. DISTRIBUTION OF LIFT, DRAG, AND PITCHING MOMENT OVER THE SPAN UNDER THE EFFECT OF PROPELLER SLIPSTREAM

1. Effective Angle of Attack

Since every airfoil section gives a zero lift at a certain direction of air flow, the angle of attack is to be measured at this lift-free flow direction. But, since on a wing of finite span, the shedding vortex band, creates interference velocities perpendicular to the direction of flow, the effective flow direction does not coincide with the direction of flight. The slipstream effect also produces a change in air-flow velocity. If α is the angle of attack with respect to the flight direction and α_i, its change due to interferences, the effective angle of attack (fig. 11) is:

$$\alpha_\infty = \alpha - \alpha_i \tag{34}$$

In the chosen coordinate system, axis x is opposite to the flight direction, axis y falls in the lateral axis, and axis z is at right angles to both downward. For the sake of clarity the term $v_{q_{virt}}$ replaces now the transverse flow component $v_{y_{virt}}$, so that the q axis lies in the plane of the virtual jet direction and the direction x.

With v_x as the axial component of the supplementary velocity due to the effect of the propeller on the wing, and $v_{z_{total}}$ as the total vertical component of the supplementary velocity due to the effect of wing and slipstream, we have:

$$\tan \alpha_i = \frac{v_{z_{total}}}{v_0 + v_x}$$

If v_x and $v_{z_{total}}$ are small relative to the flight speed v_0, it is:

$$\alpha_i = \frac{v_{z_{total}}}{v_0} \, \frac{1}{1 + \frac{v_x}{v_0}} \cong \frac{v_{z_{total}}}{v_0} - \frac{v_{z_{total}}}{v_0} \, \frac{v_x}{v_0}$$

and

$$\alpha_i = \frac{v_{z\,total}}{v_o}\left(1 - \frac{v_x}{v_o}\right) \tag{35}$$

The problem now is to define v_x and $v_{z\,total}$. For v_x the assumptions employed for slipstream in yaw may be resorted to. For the inside of the slipstream far downstream from the propeller, we find, according to figure 12:

$$v_{x_i} = v_{zus} \cos \nu$$

or, with

$$v_{zus} = s\ v_o, \quad s = s' \cos \nu$$

$$v_{x_i} = s\ v_o \cos \nu \tag{36}$$

and for small ν:

$$v_{x_i} = s\ v_o$$

At shorter distance from the propeller, the supplementary velocity is smaller. Hence we write

$$v_{x_i} = \psi_{x_i}\ s\ v_o \tag{37}$$

Quantity s follows from the coefficient of thrust loading c_s

$$s = \sqrt{1 + c_s} - 1$$

$$c_s = \frac{S}{\frac{\rho}{2} v_o^2 \frac{\pi D^2}{4}}$$

Outside of the slipstream in the remote wake of the propeller it is, according to figure 13:

$$v_{x_a} = v_{q_{virt}} \frac{s'}{r^2} \sin \nu = \frac{s'}{r^2} v_o \sin^2 \nu$$

For small angles ν, $v_{x_a} \sim 0$, hence negligible. At shorter distance from the propeller, however, its effect induces further axial velocities (reference 2) which must be written:

$$v_{x_a} = \psi_{x_a}\, s\, v_o$$

Hence equation (37) is extended in the form of

$$\frac{v_x}{v_o} = \psi_x\, s \qquad (38)$$

to include the total flow region; ψ_x is taken from figure 14.

The contributions to the vertical component $v_{z_{total}}$ come from

1. The vertical component v_ν existing as a result of the yaw at angle ν. According to figures 12 and 13, it is:

$$v_{\nu_i} = v_{zus} \sin \nu = v_o\, s\, \nu$$

$$v_{\nu_a} = -\frac{s'}{r^2} v_{q_{virt}} \cos \nu = -\frac{s}{r^2} v_o \sin \nu = -v_o\, s\, \nu \frac{1}{r^2}$$

which, written in the form

$$v_{z_I} = v_o\, s\, \nu\, \psi_\nu \qquad (39)$$

give $\psi_{\nu_i} = 1$ and $\psi_{\nu_a} = -\frac{R^2}{r^2}$, $R = 1$ being the slipstream radius. For small values ν, the distance ρ from the propeller axis may be used instead of the distance r of an outside particle of the slipstream center.

Having defined the z component of the velocity due to yaw, it may be assumed in the following on account of the assumed smallness of $\alpha_{i_{str}}$ that the slipstream is coincident with the x direction. Then: $s = s'$.

2. The vertical component of the radial velocity due to the slipstream. If R is the jet radius and r the distance of a particle from the jet axis and propeller axis, respectively, the radial velocity is (fig. 14)

$$v_r = \frac{R}{r}\, s\, v_o\, \psi_r$$

its vertical component (fig. 15) is:

$$v_{z_{II}} = s\ v_o \frac{z\ R}{r^2} \psi_r \tag{40}$$

ψ_r is taken from figure 14.

3. The vertical component of the circumferential velocity v_u due to the slipstream. The circumferential velocity caused by a vortex located on the jet axis has the value $v_u = -\frac{\Gamma_m}{2\ \pi}\frac{1}{r}$ if Γ_m is the mean slipstream twist. Its vertical component is $v_{z_{III}} = v_u \frac{y}{r}$ (fig. 16). In accord with propeller theory, it is:

$$\frac{v_u}{s\ v_o} \sim \frac{\lambda}{\eta} \frac{R}{r}$$

whence the vertical component of the circumferential velocity reads

$$v_{z_{III}} = s\ v_o \frac{\lambda}{\eta} \frac{R}{r} \frac{y}{r} = s\ v_o \frac{y\ R}{r^2} \psi_u \tag{41}$$

whereby,

$$\psi_u = 0(r > R) \quad \text{and} \quad \psi_u = \frac{\lambda}{\eta} \frac{y}{r} \ (r < R) \quad \text{respectively}$$

(λ = coefficient of advance, η = propeller efficiency).

4. The vertical component of the velocity due to the presence of singularities inside and outside the jet. It consists of

a) The induced velocity $v_{z_{IV}} = v_{i_o}(\Gamma_o)$ of the circulation distribution unaffected by the slipstream. The axes of the vortices emanating from the wing may be dealt with as falling in the x direction.

b) The vertical component of an additional flow necessary for compliance with the limiting condition, because a) alone does not satisfy the limiting conditions. For a) and b) together the equations

$$\varphi_I = \varphi_{A_a} + \varphi_{B_i} - s\ \varphi_{A_i}$$

$$\varphi_{II} = \varphi_{A_a} + \varphi_{B_i} + s\ \varphi_{B_a}$$

are applicable.

Since φ_{A_a} and φ_{B_i} are employed for a), there remain for b):

$$\left.\begin{aligned} \varphi_{I_{zus}} &= -\ s\ \varphi_{A_i} \\ \varphi_{II_{zus}} &= +\ s\ \varphi_{B_a} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (42)$$

The corresponding vertical components of the supplementary flow can be written in the form $v_{z_V} = s\ v_{i_1}(\Gamma_o)$.

Through the vertical components defined in 1, 2, 3, 4 b), the downwash conditions have changed in relation to the flow undisturbed by the slipstream. This change calls for a new circulation distribution. Since the velocities creating the new circulation distribution can be written in the form $s\ v_z$, the total circulation can be expressed with

$$\Gamma_o + s\Gamma_1$$

This supplementary circulation $s\ \Gamma_1$ induces in turn a downwash velocity. Hence, as added vertical component:

5. The supplementary induced velocity

$$v_{z_{VI}} = s\ v_{i_o}(\Gamma_1)$$

due to the supplementary circulation distribution.

But then, compliance with the limiting conditions would necessitate the reflection of $s\ \Gamma_1$. According to equations (42), the new supplementary potentials would now have the factor s^2, which, however, in accord with the assumed smallness of s, may be neglected. The analysis can therefore be broken off with $s v_{i_o}(\Gamma_1)$. For, on assuming that through the supplementary velocities and the thereby necessary reflections, the total circulation is

$$\Gamma = \Gamma_0 + s\,\Gamma_1 + s^2\,\Gamma_2 + s^3\,\Gamma_3 + \ldots\ldots\ldots$$

we find for

$$|\Gamma_\nu| < |\Gamma_0| \quad (\nu = 1.2, \ldots\ldots n)$$

with

$$\Gamma^* = \Gamma_0\,(1 + s + s^2 + \ldots\ldots) = \Gamma_0\,\frac{1}{1 - s}$$

i.e., a circulation greater than the total circulation Γ. The difference between Γ^* and $\Gamma_0 + s\,\Gamma_1$ is then approximately

$$\Gamma^* - (\Gamma_0 + s\Gamma_1) \sim \Gamma^* - (\Gamma_0 + s\Gamma_0) = \Gamma_0\left[\frac{1}{1 - s} - (1 + s)\right]$$

$$= \Gamma_0\,\frac{s^2}{1 - s}$$

The omission caused by the interruption of the series is of the order of magnitude of $s^2\,\Gamma_0$.

Then the sum of all vertical components gives:

$$\frac{v_{z_{total}}}{v_0} = \frac{v_{i_0}(\Gamma_0)}{v_0} + s\left[\frac{v_{i_1}(\Gamma_0)}{v_0} + \frac{v_{i_0}(\Gamma_1)}{v_0} + \nu\,\psi_\nu + \frac{z\,R}{r^2}\,\psi_r + \frac{y\,R}{r^2}\,\psi_u\right]$$

$$= \frac{v_{z_0}}{v_0} + s\,\frac{v_{z_1}}{v_0}$$

According to equation (35) we have

$$\alpha_i = \frac{v_{z_{total}}}{v_0}\,(1 - s\,\psi_x)$$

and, when disregarding the terms of higher order:

$$\alpha_i = \frac{v_{z_0}}{v_0} + s\,\frac{v_{z_1}}{v_0} - s\,\psi_x\,\frac{v_{z_0}}{v_0} = \frac{v_{z_0}}{v_0} + s\left(\frac{v_{z_1}}{v_0} - \psi_x\,\frac{v_{z_0}}{v_0}\right)$$

which, with the ascertained values, gives:

$$\alpha_i = \frac{v_{i_o}(\Gamma_o)}{v_o} + s\left[\frac{v_{i_1}(\Gamma_o)}{v_o} + \frac{v_{i_o}(\Gamma_1)}{v_o} + \nu\,\psi_\nu \right.$$

$$\left. + \frac{z\ R}{r^2}\,\psi_r + \frac{y\ R}{r^2}\,\psi_u - \frac{v_{i_o}(\Gamma_o)}{v_o}\,\psi_x\right] \qquad (43)$$

2. The Supplementary Circulation

The circulation developed under the effect of the slipstream is, according to the foregoing,

$$\Gamma = \Gamma_o + s\,\Gamma_1$$

Here Γ_o is the circulation existing without slipstream effect, but with the same degree of flow turbulence as if the slipstream were present. $\Delta\Gamma = s\,\Gamma_1$ is the supplementary circulation due to the effect of the slipstream. The solution of Γ_o is known from airfoil theory and follows from

$$\Gamma_o = \pi\ v_o\ t_{red}\left(\alpha - \frac{v_{i_o}(\Gamma_o)}{v_o}\right)$$

with

$$t_{red} = t\,\frac{d\ c_a/d\ \alpha_\infty}{2\ \pi}$$

The total circulation is:

$$\Gamma = \pi\ t_{red}\ (v_o + v_x)(\alpha - \alpha_i) = \pi\ t_{red}\ v_o\ (1 + s\,\psi_x)(\alpha - \alpha_i)$$

Hence:

$$\frac{\Gamma}{\pi\ v_o\ t_{red}} = \frac{\Gamma_o}{\pi\ v_o\ t_{red}} + \frac{s\ \Gamma_1}{\pi\ v_o\ t_{red}} = (1 + s\psi_x)(\alpha - \alpha_i)$$

and, by omission of the terms of higher order:

$$\frac{\Gamma_o}{\pi\ v_o\ t_{red}} + \frac{s\ \Gamma_1}{\pi\ v_o\ t_{red}} = \left[\alpha - \frac{v_{i_o}(\Gamma_o)}{v_o}\right] - s\left[\frac{v_{i_1}(\Gamma_o)}{v_o} + \frac{v_{i_o}(\Gamma_1)}{v_o}\right.$$

$$\left. + \nu\,\psi_\nu + \frac{z\ R}{r^2}\,\psi_r + \frac{y\ R}{r^2}\,\psi_u - \frac{v_{i_o}(\Gamma_o)}{v_o}\,\psi_x - \frac{\Gamma_o}{\pi\ v_o\ t_{red}}\,\psi_x\right]$$

which leaves, as supplementary circulation:

$$\frac{\Gamma_1}{\pi\ v_o\ t_{red}} = \left(\frac{v_{i_o}(\Gamma_o)}{v_o} + \frac{\Gamma_o}{\pi\ v_o\ t_{red}}\right)\psi_x - \frac{v_{i_1}(\Gamma_o)}{v_o}$$

$$- \frac{v_{i_o}(\Gamma_1)}{v_o} - \nu\ \psi_\nu - \frac{z\ R}{r^2}\ \psi_r - \frac{y\ R}{r^2}\ \psi_u$$

In this manner the determination of the circulation reduces to the circulation distribution of a wing of equal chord distribution t_{red} and a geometrical angle of attack distribution

$$\alpha_1 = \left(\frac{v_{i_o}(\Gamma_o)}{v_o} + \frac{\Gamma_o}{\pi\ v_o\ t_{red}}\right)\psi_x$$

$$- \frac{v_{i_1}(\Gamma_o)}{v_o} - \nu\ \psi_\nu - \frac{z\ R}{r^2}\ \psi_r - \frac{y\ R}{r^2}\ \psi_u$$

$$= \alpha\ \psi_x - \frac{v_{i_1}(\Gamma_o)}{v_o} - \nu\ \psi_\nu - \frac{z\ R}{r^2}\ \psi_r - \frac{y\ R}{r^2}\ \psi_u$$

in homogeneous flow velocity v_o.

Since the individual flow contributions are linearly superposable, Γ_1, itself can be obtained by linear superposition of the contributions to Γ_1 due to individual flow contributions. Thus the prediction of Γ_1 and of the individual parts reduces to the well-known problem of airfoil theory, for the solution of which practical calculating methods have been developed but which need not be discussed here.*

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*Although the problem of slipstream effect on an airplane wing has been successfully reduced to the well-known problem of circulation about a wing in homogeneous flow, it still appears advisable to try this method on a number of illustrated examples, but for which we lacked the necessary time. During the preparations for these examples, it was found that the substantial effects can even be studied on the example of a wing of constant chord and infinite span. Koning himself reports some results for this case. We succeeded in presenting some of these results in closed form and subsequent studies are to be extended to include calculation of downwash and behavior of the control surfaces.

3. The Supplementary Lift Distribution

The lift proportion of a wing element is:

$$d\ A = \rho\ \Gamma\ v_\infty\ dy$$

where v_∞ is the flow velocity

$$v_\infty = v_0 + v_x = v_0(1 + s\ \psi_x)$$

With $\Gamma = \Gamma_0 + s\ \Gamma_1$, it then affords

$$d\ A = \rho(\Gamma_0 + s\Gamma_1)\ v_0(1 + s\ \psi_x)dy$$

or, neglecting terms of higher order:

$$d\ A = [\rho\ \Gamma_0\ v_0 + s\ \rho\ v_0(\Gamma_1 + \psi_x\ \Gamma_0)]\ dy$$

Since the lift without slipstream effect is given by

$$d\ A_0 = \rho\ \Gamma_0\ v_0\ dy$$

the change in lift distribution due to slipstream effect is

$$d\ A - d\ A_0 = d\ \Delta\ A = s\ \rho\ v_0(\Gamma_1 + \psi_x\ \Gamma_0)\ dy$$

Putting $d\ \Delta\ A = s\ d\ A_1$, we have:

$$d\ A_1 = \rho\ v_0(\psi_x\ \Gamma_0 + \Gamma_1)dy \qquad (45)$$

The proportionate change of lift is given through

$$\frac{\frac{d}{dy}\Delta\ A}{s\ \frac{d}{dy}A_0} = \frac{\frac{d}{dy}A_1}{\frac{d}{dy}A_0} = \frac{\psi_x\ \Gamma_0 + \Gamma_1}{\Gamma_0} = \psi_x + \frac{\Gamma_1}{\Gamma_0}$$

4. The Supplementary Drag Distribution

The drag of an element of a wing is

$$d\ W = \rho\ \Gamma\ \alpha_i\ v_\infty\ dy + \frac{\rho}{2}v_\infty^2\ t\ dy\ c_{w_p}$$

With

$$\alpha_i \; v_\infty = \frac{v_{z\,total}}{v_o + v_x}\,(v_o + v_x) = v_{z\,total} = v_o \left[\frac{v_{i_o}(\Gamma_o)}{v_o} \right.$$

$$\left. + \; s\left\{ \frac{v_{i_1}(\Gamma_o)}{v_o} + \frac{v_{i_o}(\Gamma_1)}{v_o} + \upsilon\,\psi_\upsilon + \frac{z\,R}{r^2}\,\psi_r + \frac{y\,R}{r^2}\,\psi_u \right\} \right]$$

if affords

$$d\,W = \rho(\Gamma_o + s\Gamma_1)\left[\frac{v_{i_o}(\Gamma_o)}{v_o} + s\left\{ \frac{v_{i_1}(\Gamma_o)}{v_o} + \frac{v_{i_o}(\Gamma_1)}{v_o} + \upsilon\,\psi_\upsilon \right.\right.$$

$$\left.\left. + \frac{z\,R}{r^2}\,\psi_r + \frac{y\,R}{r^2}\,\psi_u \right\} \right] v_o\,dy + \frac{\rho}{2}\,(v_o + v_x)^2 t\,dy\,c_{w_p}$$

or, neglecting terms of higher order:

$$d\,W = \left[\rho\,\Gamma_o\,v_o\,\frac{v_{i_o}(\Gamma_o)}{v_o}\,dy + \frac{\rho}{2}\,v_o^2\,t\,dy\,c_{w_p} \right]$$

$$+ \; s\,\rho\left[\Gamma_1\,\frac{v_{i_o}(\Gamma_o)}{v_o} + \Gamma_o\left\{ \frac{v_{i_1}(\Gamma_o)}{v_o} + \frac{v_{i_o}(\Gamma_1)}{v_o} + \upsilon\,\psi_\upsilon \right.\right.$$

$$\left.\left. + \frac{z\,R}{r^2}\,\psi_r + \frac{y\,R}{r^2}\psi_u \right\} v_o + v_o^2\,\psi_x\,t\,c_{w_p} \right] dy$$

As the drag of the wing without slipstream effect would be given by:

$$d\,W_o = \rho\,\Gamma_o\,v_{i_o}(\Gamma_o)\,dy + \frac{\rho}{2}\,v_o^2\,t\,dy\,c_{w_p}$$

the supplementary drag amounts to

$$d\,W - d\,W_o = d\,\Delta\,W = s\left[\rho\Gamma_o v_o\,dy \left\{ \frac{\Gamma_1}{\Gamma_o}\,\frac{v_{i_o}(\Gamma_o)}{v_o} + \frac{v_{i_1}(\Gamma_o)}{v_o} \right.\right.$$

$$\left.\left. + \frac{v_{i_o}(\Gamma_1)}{v_o} + \upsilon\psi_\upsilon + \frac{z\,R}{r^2}\,\psi_r + \frac{y\,R}{r^2}\,\psi_u \right\} + \rho\,v_o^2\,\psi_x\,t\,c_{w_p}\,dy \right]$$

Putting

$$d\,\Delta W = d\,\Delta W_i + d\,\Delta W_p = s\,d\,W_1$$

where the subscripts i and p present the induced and the profile drag, respectively, we have:

$$d\,W_1 = \rho\,\Gamma_0\,v_0\,dy\left[\frac{\Gamma_1}{\Gamma_0}\,\frac{v_{i_0}(\Gamma_0)}{v_0} + \frac{v_{i_1}(\Gamma_0)}{v_0} + \frac{v_{i_0}(\Gamma_1)}{v_0}\right.$$

$$\left. + \nu\,\psi_\nu + \frac{z\,R}{r^2}\,\psi_r + \frac{z\,R}{r^2}\,\psi_u\right] + \rho\,v_0{}^2\,\psi_x\,t\,c_{w_p}\,dy \qquad (46)$$

and for the proportionate change in induced drag

$$\frac{\frac{d}{dy}\,\Delta\,W_i}{s\,\frac{d}{dy}\,W_{i_0}} = \frac{\frac{d}{dy}\,W_{i_1}}{\frac{d}{dy}\,W_{i_0}} = \frac{v_0}{v_{i_0}(\Gamma_0)}$$

$$\left[\frac{\Gamma_1}{\Gamma_0}\,\frac{v_{i_0}(\Gamma_0)}{v_0} + \frac{v_{i_1}(\Gamma_0)}{v_0} + \frac{v_{i_0}(\Gamma_1)}{v_0} + \nu\,\psi_\nu + \frac{z\,R}{r^2}\,\psi_r + \frac{y\,R}{r^2}\,\psi_u\right] =$$

$$= \frac{v_{i_1}(\Gamma_0) + v_{i_0}(\Gamma_1)}{v_{i_0}(\Gamma_0)} + \frac{v_0}{v_{i_0}(\Gamma_0)}\left(\nu\,\psi_\nu + \frac{z\,R}{r^2}\,\psi_r + \frac{y\,R}{r^2}\,\psi_u\right) + \frac{\Gamma_1}{\Gamma_0}$$

The proportionate change in profile drag is

$$\frac{\frac{d}{dy}\,\Delta\,W_p}{s\,\frac{d}{dy}\,W_{p_0}} = \frac{\frac{d}{dy}\,W_{p_1}}{\frac{d}{dy}\,W_{p_0}} = 2\,\psi_x$$

The change in drag accordingly consists of three contributions the causes of which are:

1. The increment $s\,\psi_x\,v_0$ of the flow velocity (change of profile drag).

2. The change $s\,\Gamma_1$ of the circulation (last proportions to the change in induced drag).

3. The change $s\,v_z$ in the downward component of the effective flow velocity (the remaining parts of the change in induced drag).

5. The Distribution of the Supplementary Pitching Moment

Every airfoil section has one reference point F, for which the moment coefficient c_{m_F} is constant in at least one certain angle of attack range. The pitching moment of a wing particle is referred to this point F:

$$d\,M_F = \frac{\rho}{2}\,v_\infty^2\,t^2\,dy\,c_{m_F}$$

In view of $v_\infty = v_o(1 + s\,\psi_x)$, the omission of terms of higher order leaves

$$d\,M_F = \frac{\rho}{2}\,v_o^{\,2}\,t^2\,dy\,c_{m_F} + s\,\frac{\rho}{2}\,v_o^{\,2}\,t^2\,dy\,c_{m_F}\,2\,\psi_x$$

$$= \frac{\rho}{2}\,v_o^{\,2}\,t^2\,dy\,c_{n_F}(1 + 2\,s\,\psi_x)$$

Without slipstream effect, the pitching moment would be

$$d\,M_{F_o} = \frac{\rho}{2}\,v_o^{\,2}\,t^2\,dy\,c_{m_F}$$

hence the change in pitching moment is:

$$d\,\Delta\,M_F = \frac{\rho}{2}\,v_o^{\,2}\,t^2\,dy\,c_{n_F}\,2\,s\,\psi_x = s\,d\,M_{F_1} \qquad (47)$$

and the proportionate change in pitching moment becomes:

$$\frac{\frac{d}{dy}\,\Delta\,M_F}{s\,\frac{d}{dy}\,M_{F_o}} = \frac{\frac{d}{dy}\,M_{F_1}}{\frac{d}{dy}\,M_{F_o}} = 2\,\psi_x$$

But, according to this, the determination of the total pitching moment is contingent upon the reference points F being located on a straight line perpendicular to the direction of flight. Suppose this is not the case and F lies by x_F behind a fixed reference point. Then

$$d\,M = x_F\,d\,A + d\,M_F$$

is referred to this new point, and with the moment of the undisturbed wing

$$d\,M_0 = x_F\,d\,A_0 + d\,M_{F_0}$$

the change due to the slipstream

$$d\,\Delta\,M = s\,d\,M_1 = s(x_F\,d\,A_1 + d\,M_{F_1})$$

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

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2. Weinig, F.: Die Strömungsverhältnisse im Schraubenstrahl. Jahrbuch 1936 der Lilienthal-Gesellschaft fur Luftfahrtforschung, pp. 29-56.

General case where slipstream is at angle of yaw with free stream.

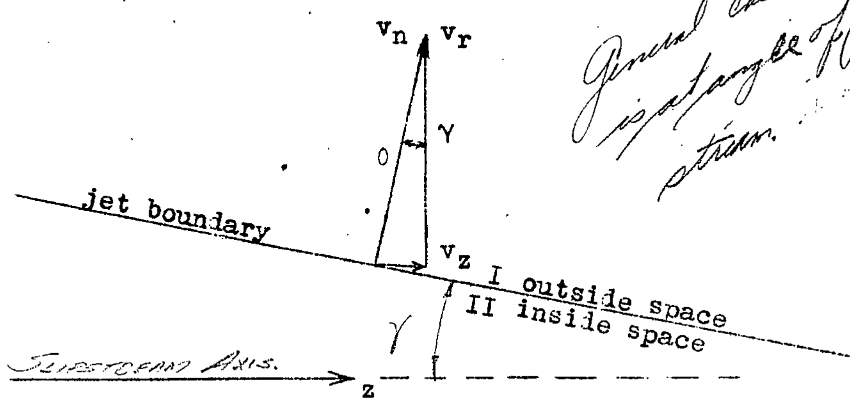

Figure 1.- The vanishing normal component of the velocity at the jet boundary.

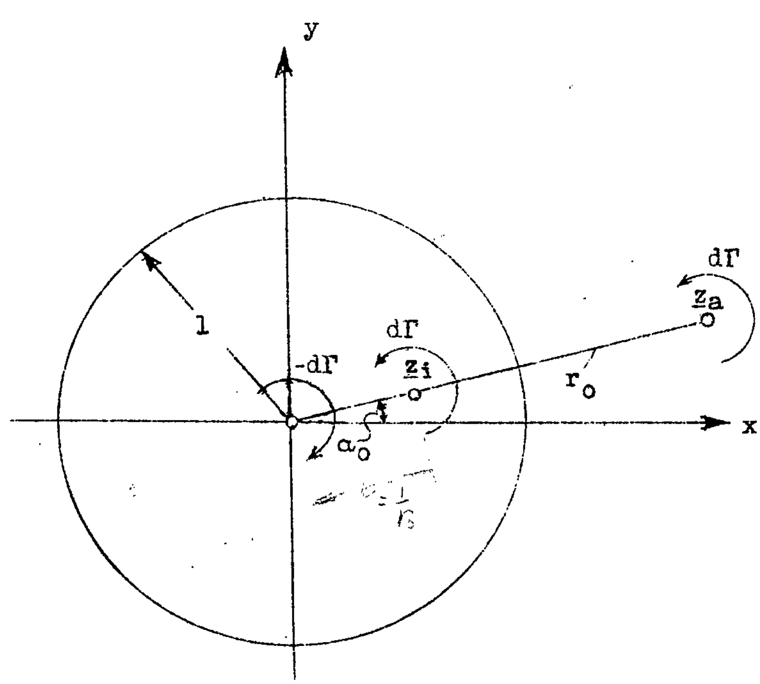

Figure 2.- Reflection of a vortex on the circle.

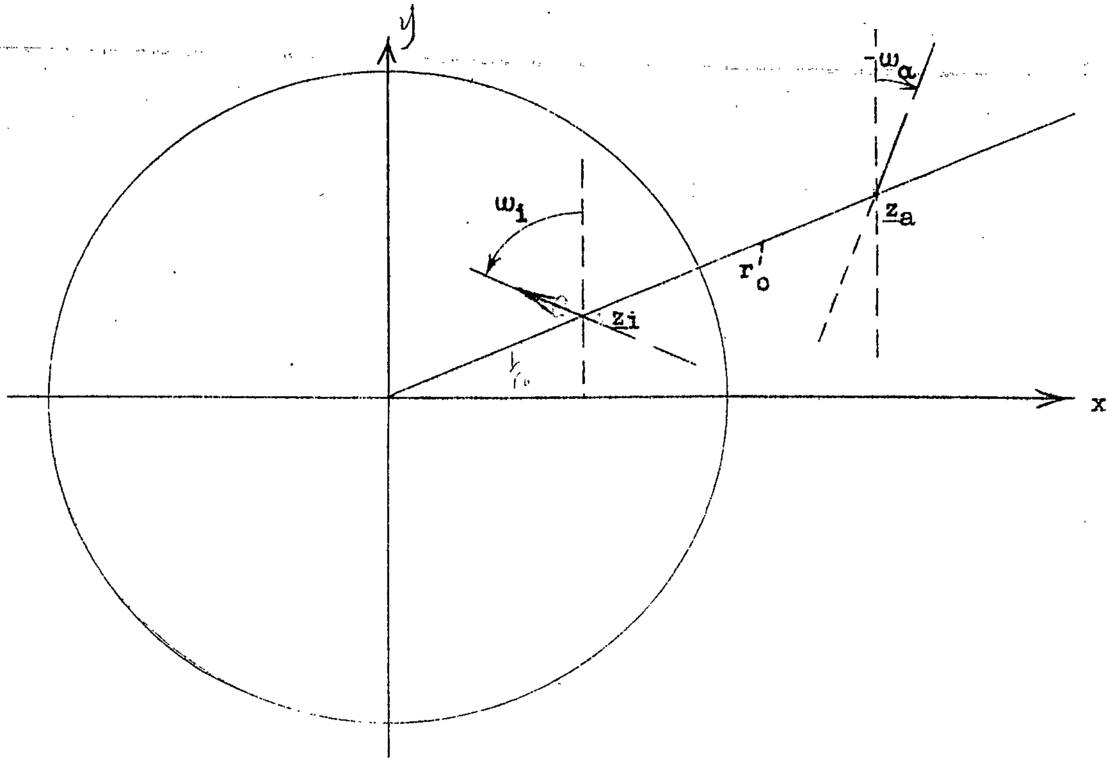

Figure 3.- Reflection of a doublet on the circle.

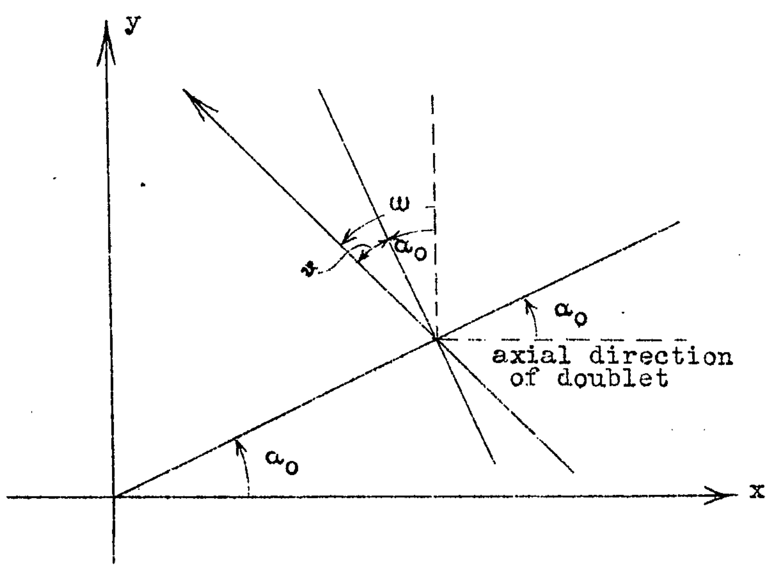

Figure 4.- Slope of axis of doublet towards positive axis x , positive slope in counter-clockwise direction.

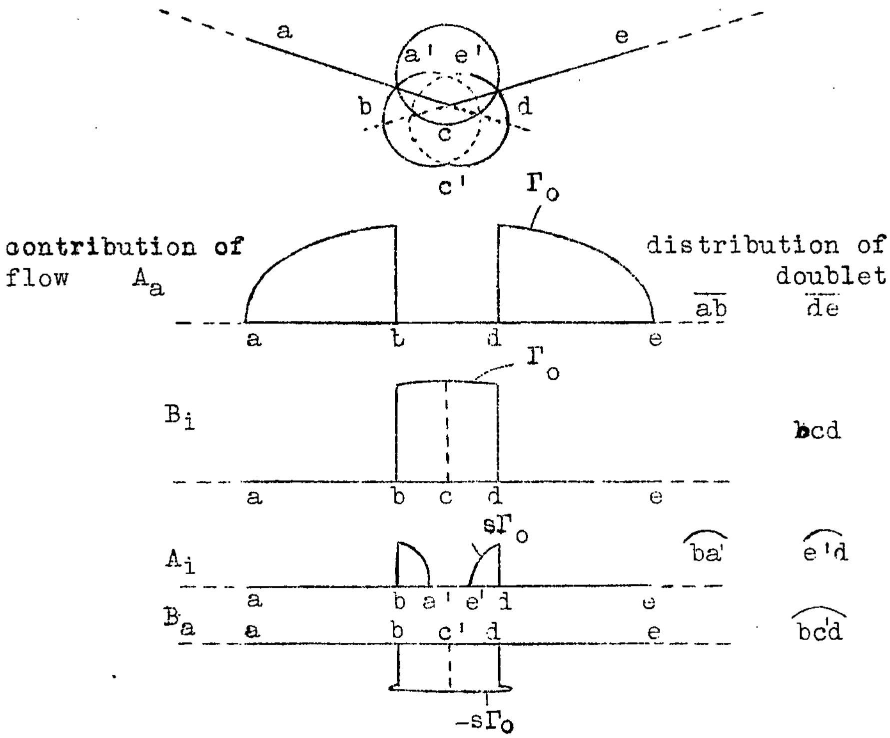

Figure 5.- Reflection of the outline of a wing with dihedral on the circle, and the interference contributions given by limiting conditions for elliptic lift distribution.

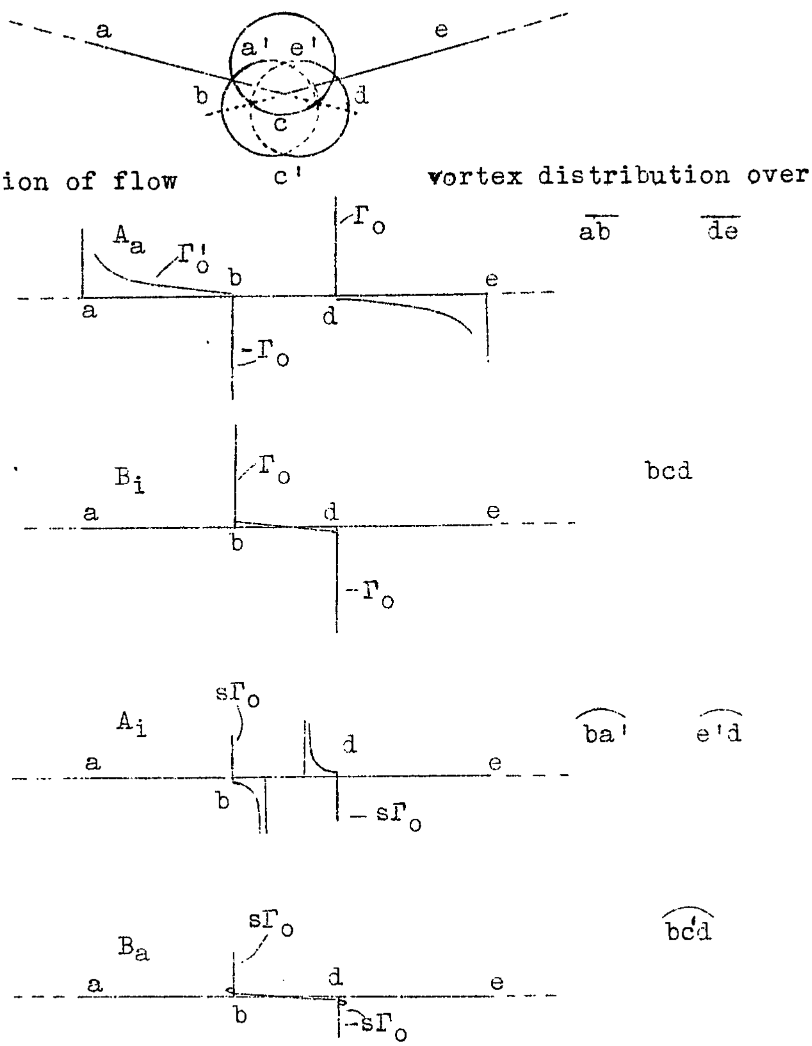

Figure 6.- The vortex distributions corresponding to the flow contributions for elliptic lift distribution.

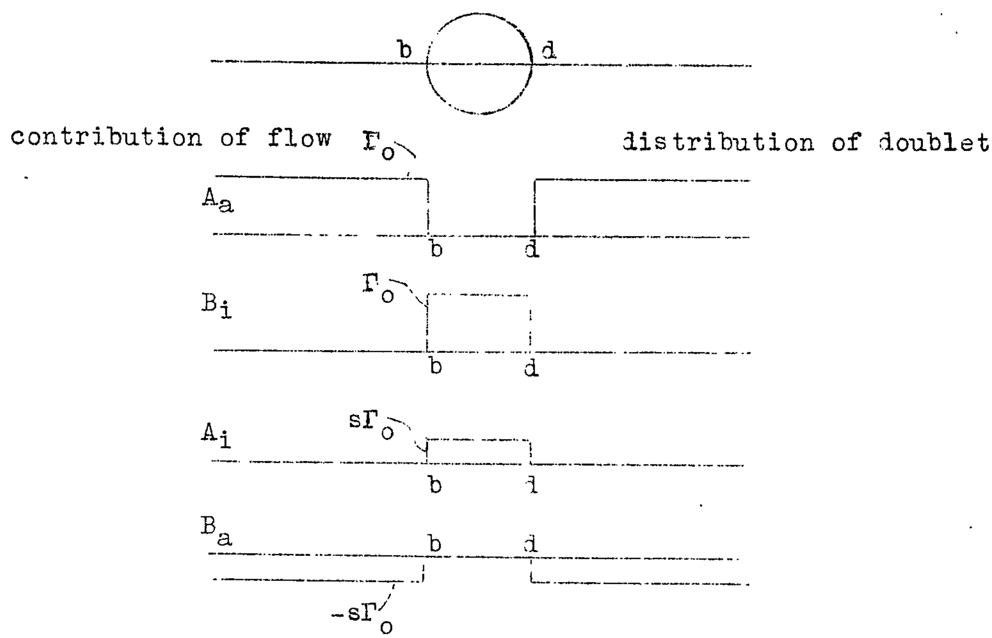

Figure 7.- Reflection of outline of a wing of constant chord and infinite span, and the flow contributions given by the limiting conditions.

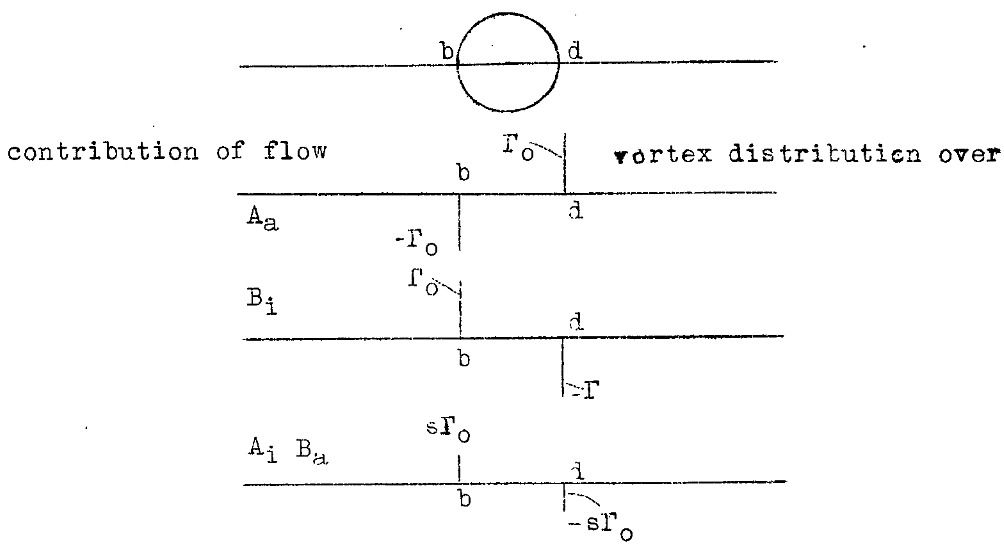

Figure 8.- The vortex distributions corresponding to the flow contributions for constant lift distribution.

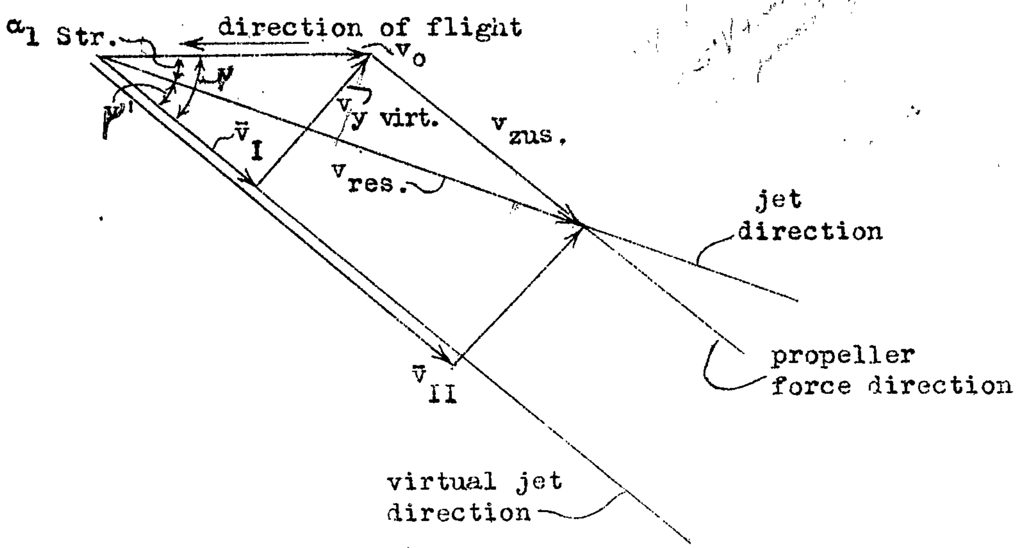

Figure 9.- The velocities within a yawed slipstream.

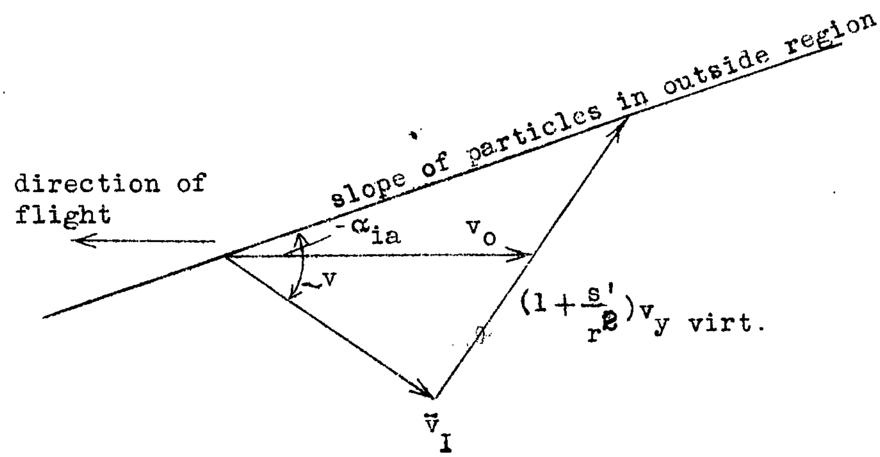

Figure 10.- The velocities outside of a yawed slipstream.

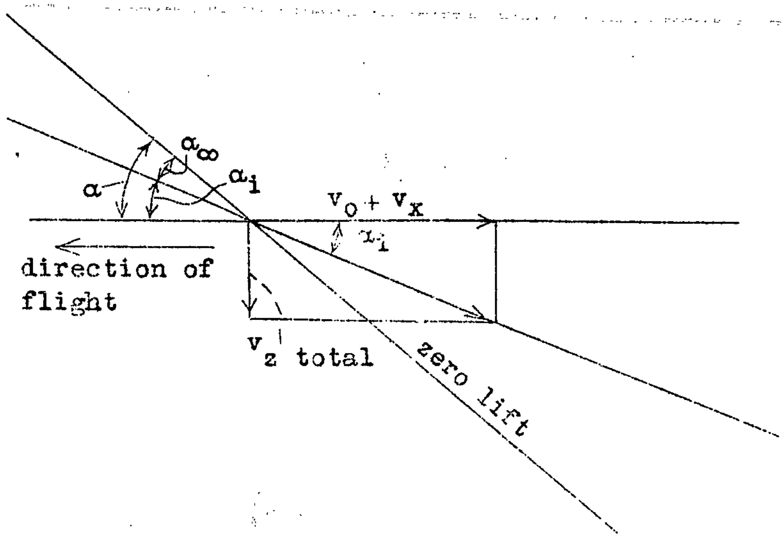

Figure 11.- The effective angle of attack.

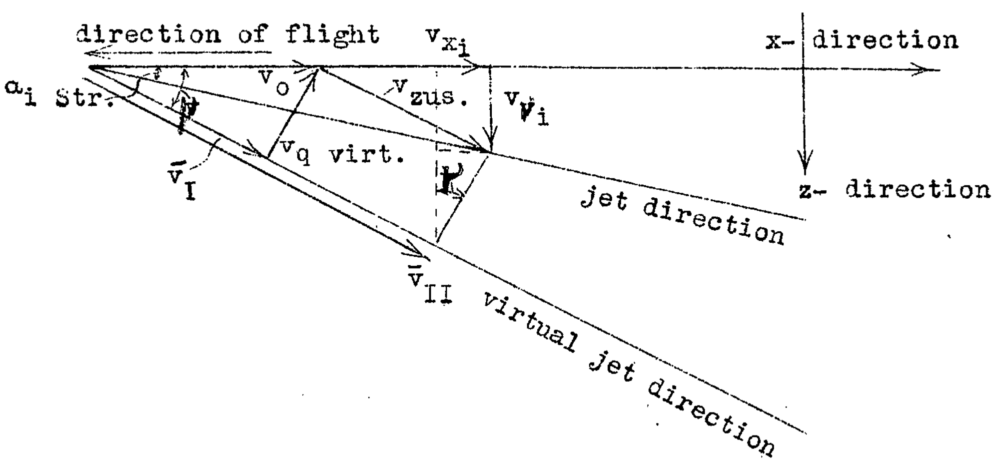

Figure 12.- The supplementary velocities inside and outside of the slipstream.

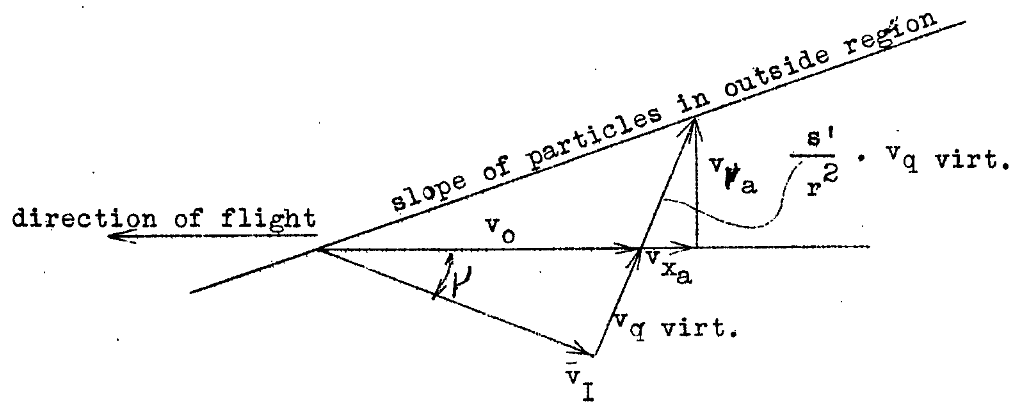

Figure 13.- The supplementary velocities inside and outside of the slip-stream.

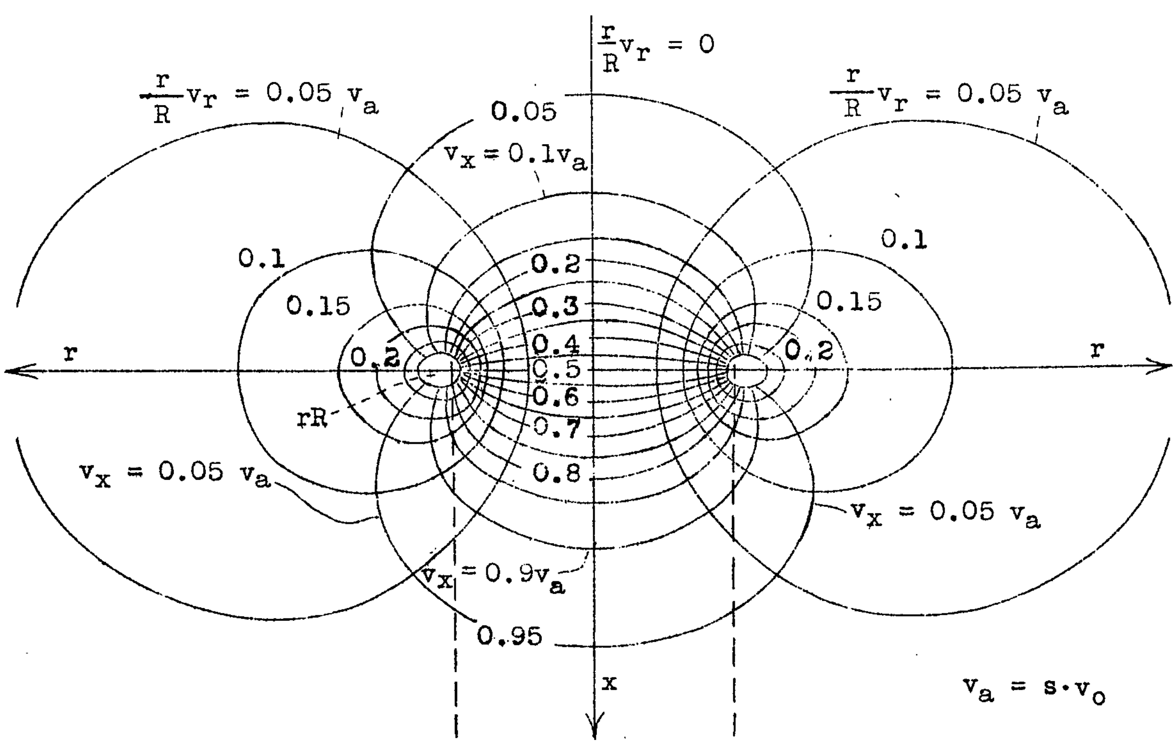

Figure 14.- Lines of equal axial components v_x and of the r-times radial component v_r of the interference due to a propeller at constant thrust grading.

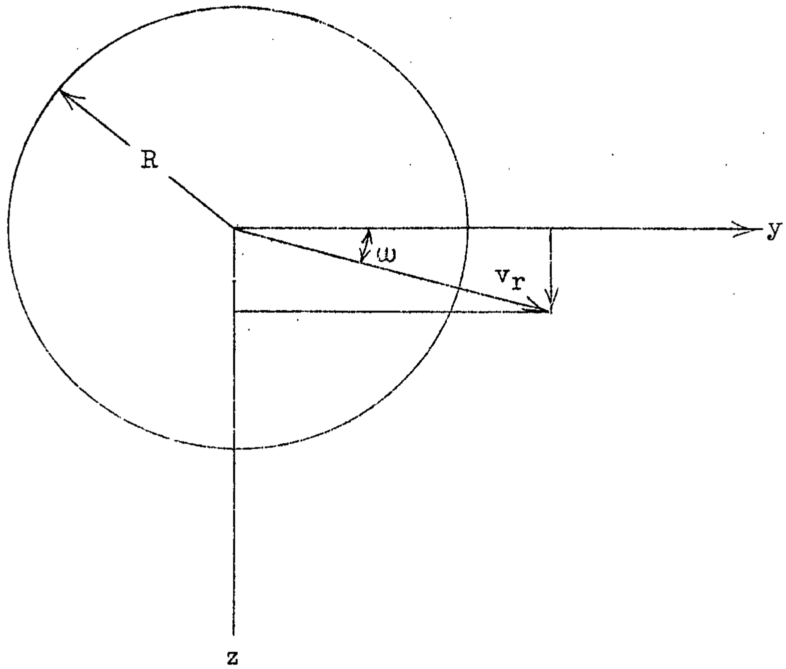

Figure 15.- The vertical component of the radial velocity.

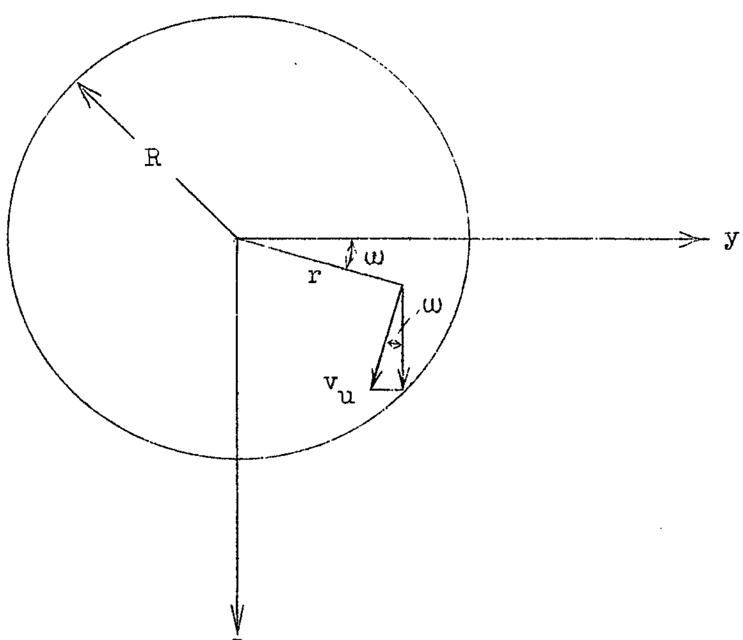

Figure 16.- The vertical component of the velocities for constant slipstream twist.